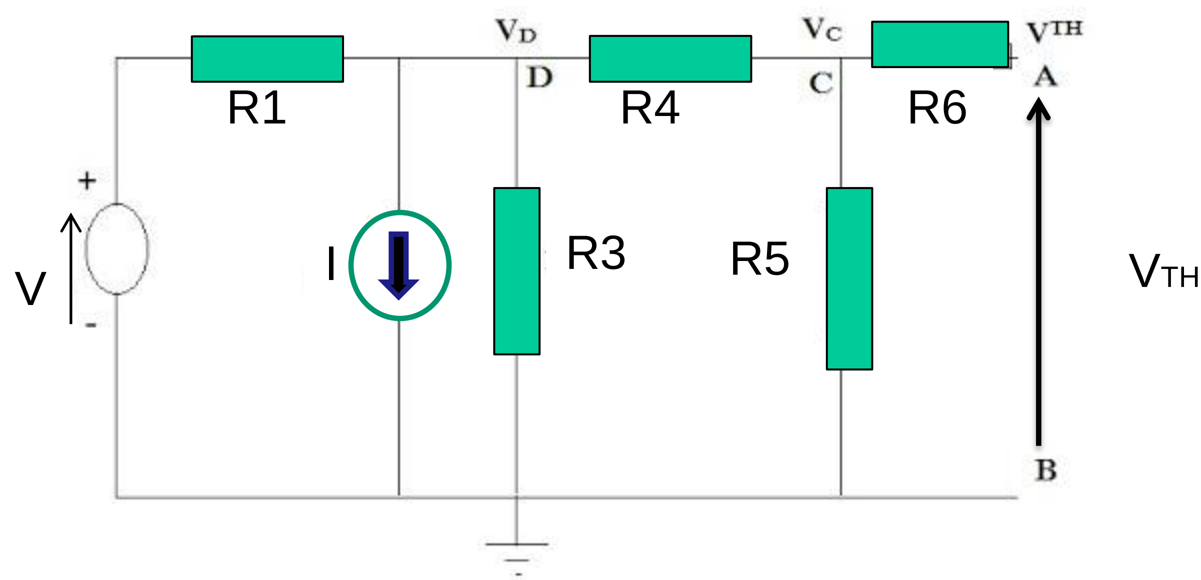
Final exam
Solutions

Problem 1 (2 points)

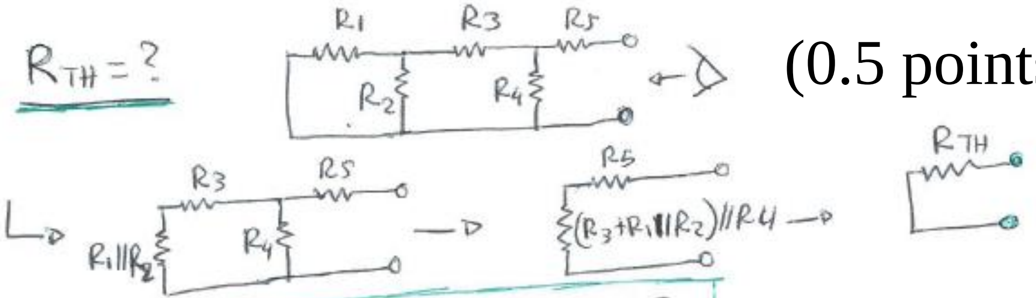


(a: 1.5 points) Give the Thévenin equivalent between A and B by calculating V_{TH} (1 point) and R_{TH} (0.5 points) using only the Thevenin-Norton concepts to analyze this problem.

(b: 0.5 points) If you connect a resistor R_L across the terminals A and B then calculate the current that flows through this resistor.

Solution

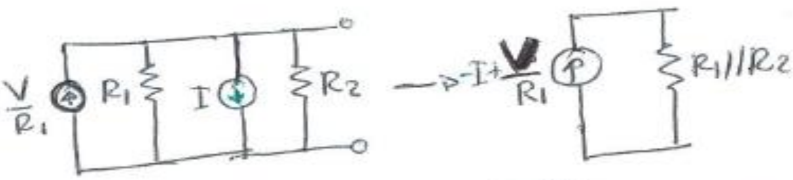
(a) $R_{TH} = ?$ (0.5 points)



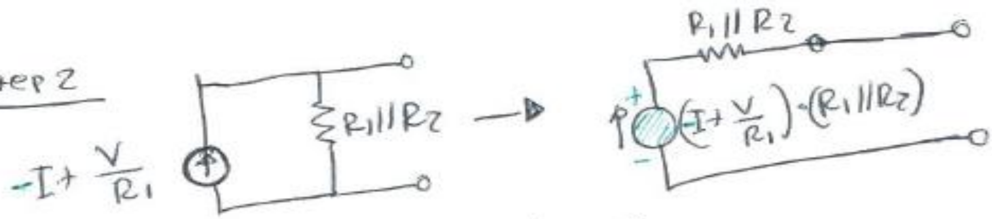
$$R_{TH} = R_5 + R_4 // [R_3 + (R_1 // R_2)]$$

$V_{TH} = ?$ (1 point)

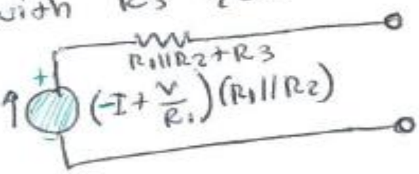
Step 1 :



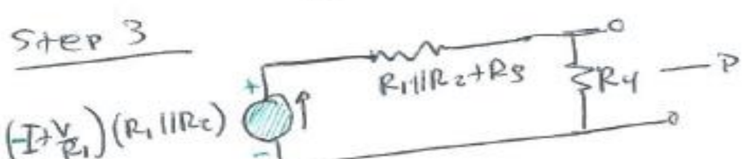
Step 2

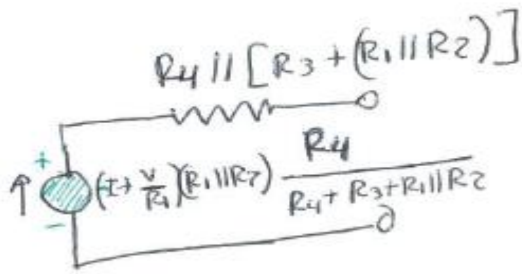


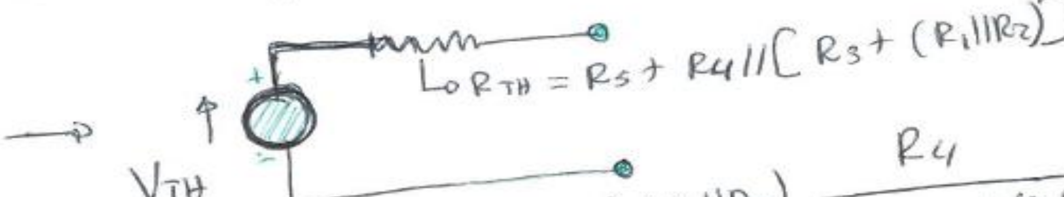
Together with R3 you have



Step 3





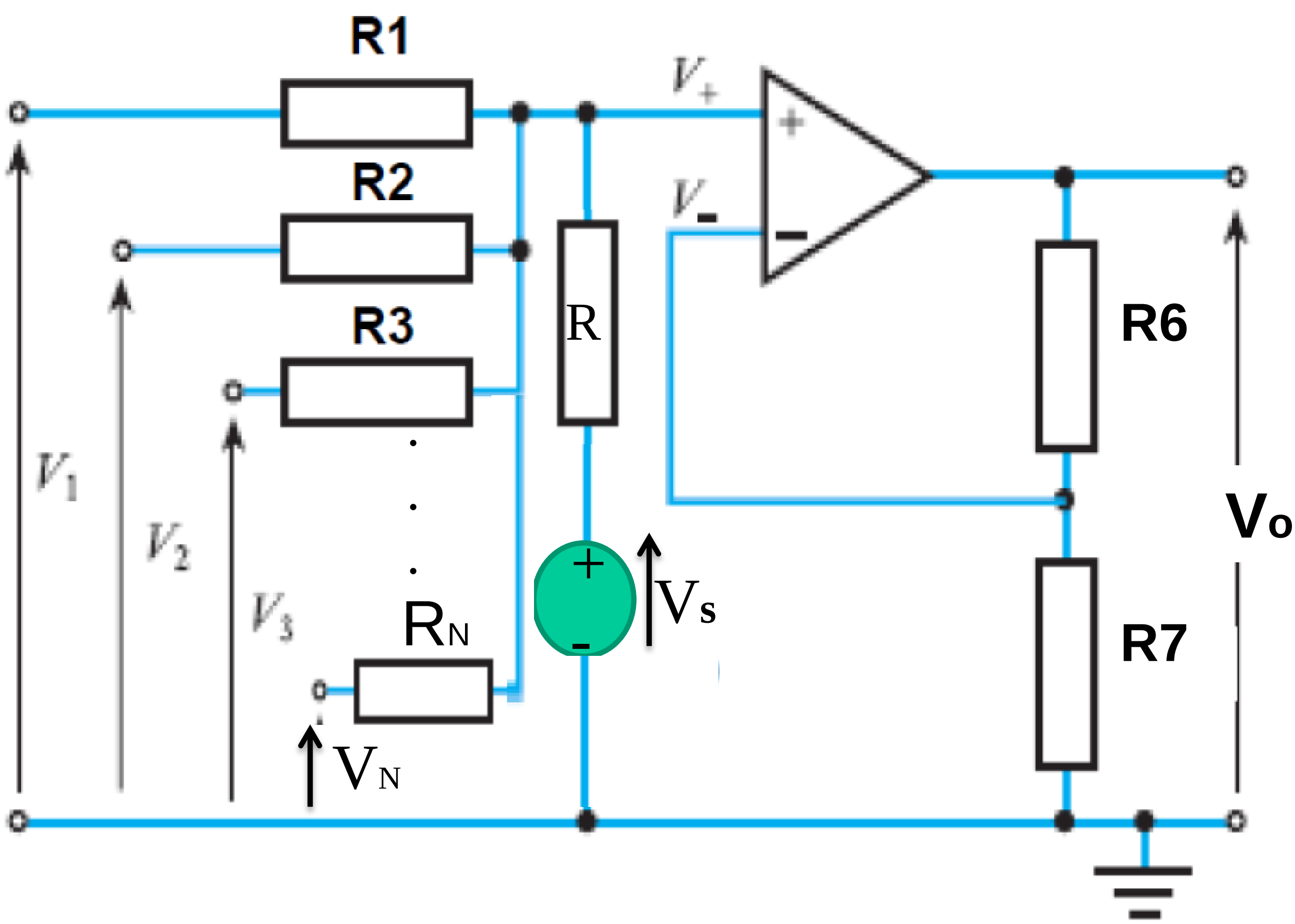


$$V_{TH} = \left(-I + \frac{V}{R_1}\right)(R_1 // R_2)\,\frac{R_4}{R_4 + R_3 + (R_1 // R_2)}$$

(b) Current through RL: (0.5 points) $I_L = V_{TH}\,[1/(R_L + R_{TH})]$

Problem 2 (2 points)

Ideal opamp: V+=V-



Consider a circuit with input of N voltage sources V_i (i=1, 2, …N). The resistor R is connected to the input of the ideal opamp via an additional voltage source Vs.

(a) Calculate the potential V_+ (1 point)

(b) Calculate the output voltage V_o as a functions of V_i's, V_s, and the resistors of the circuit shown above (1 point).

Solution

(a) The sum of the currents via V_i's equals the current that flows via R (K-law 1):

$$\sum_{i=1}^{N} I_i = I_R \quad (1) \qquad I_i = \frac{V_i - V_+}{R_i} \quad (2)$$

$$I_R = \frac{V_+ - V_S}{R} \quad (3)$$

$$(1,2,3) \Rightarrow \sum_{i=1}^{N} \frac{V_i - V_+}{R_i} = \frac{V_+ - V_S}{R} \Rightarrow$$

$$V_+ \left(\sum_{i=1}^{N} \frac{1}{R_i} + \frac{1}{R} \right) = \sum_{i=1}^{N} \frac{V_i}{R_i} + \frac{V_S}{R} \Rightarrow$$

$$\boxed{V_+ = \frac{\sum_{i=1}^{N} \frac{V_i}{R_i} + \frac{V_S}{R}}{\sum_{i=1}^{N} \frac{1}{R_i} + \frac{1}{R}}} \quad (4)$$

(b) $V_+ = V_-$, $V_- = V_0 \frac{R_7}{R_7 + R_6}$ (5)

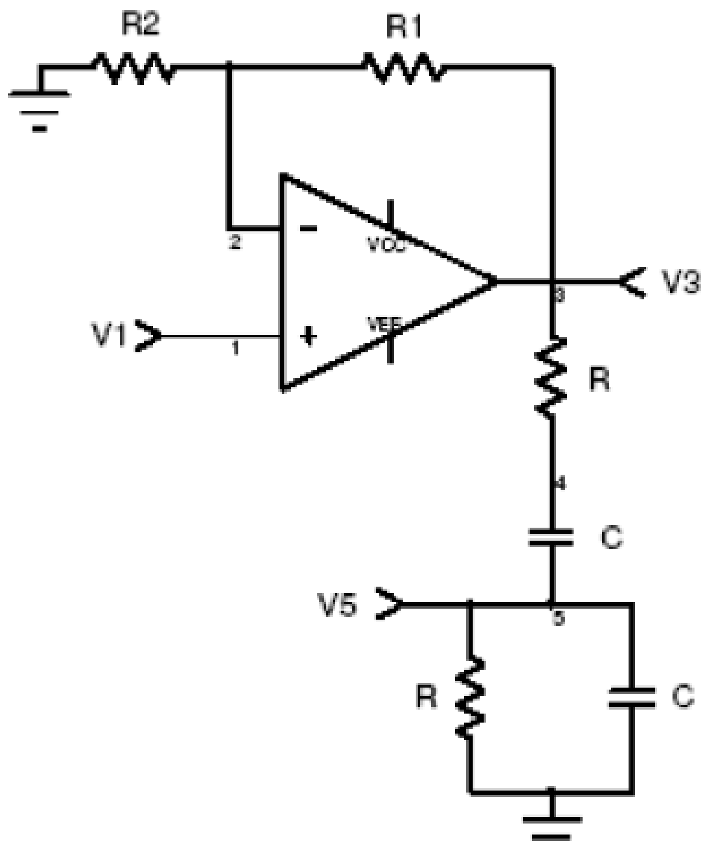
From (4) & (5) since $V_+ = V_-$ for ideal opamp we obtain

$$V_0 \frac{R_7}{R_7 + R_6} = \frac{\sum_{i=1}^{N} \frac{V_i}{R_i} + \frac{V_S}{R}}{\sum_{i=1}^{N} \frac{1}{R_i} + \frac{1}{R}} \Rightarrow$$

$$\boxed{V_0 = \left(1 + \frac{R_6}{R_7}\right) \frac{\sum_{i=1}^{N} \frac{V_i}{R_i} + \frac{V_S}{R}}{\sum_{i=1}^{N} \frac{1}{R_i} + \frac{1}{R}}}$$

Problem 3 (1.5 points)

Consider the circuit (Wien bridge oscillator):

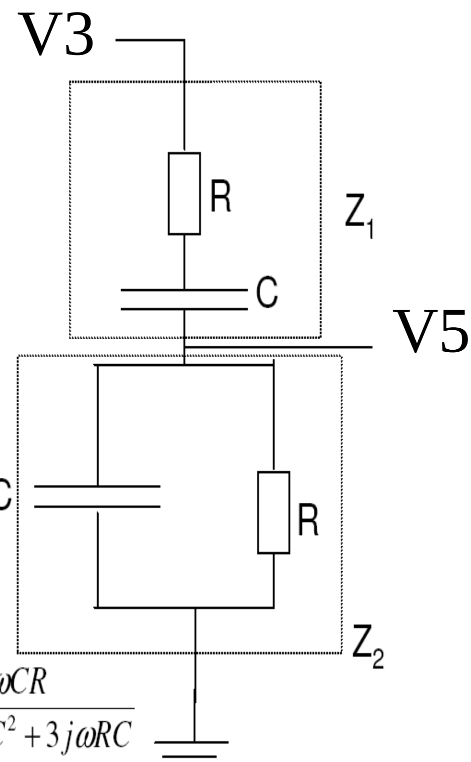


(a) Calculate the transfer ratio A=V3/V1 (V+=V-; 0.5 points)
(b) Calculate the transfer ratio B=V5/V3. For what value of ωRC is B real (0.5 points)?
(c) For what value of R_1 / R_2 is $AB = 1$? (0.5 points).

Solution

(a) V+=V1, V-=V3{R2/[R2+R1]} (voltage divider)
V+=V- → V1= V3{R2/R2+R1} → A=V3/V1=1+(R1/R2)

(b) In the more general case we have:



$$\frac{V5}{V3} = \frac{Z_2}{Z_1+Z_2}$$

$$Z_1 = R + \frac{1}{j\omega C} = \frac{j\omega RC+1}{j\omega C}$$

$$\frac{1}{Z_2} = \frac{1}{R} + j\omega C = \frac{1+j\omega RC}{R} \to Z_2 = \frac{R}{1+j\omega RC}$$

$$\frac{V5}{V3} = \frac{\frac{R}{1+j\omega RC}}{\frac{j\omega RC+1}{j\omega C}+\frac{R}{1+j\omega RC}} = \frac{R}{\frac{1+2j\omega RC-\omega^2R^2C^2}{j\omega C}+R} = \frac{j\omega CR}{1-\omega^2R^2C^2+3j\omega RC}$$

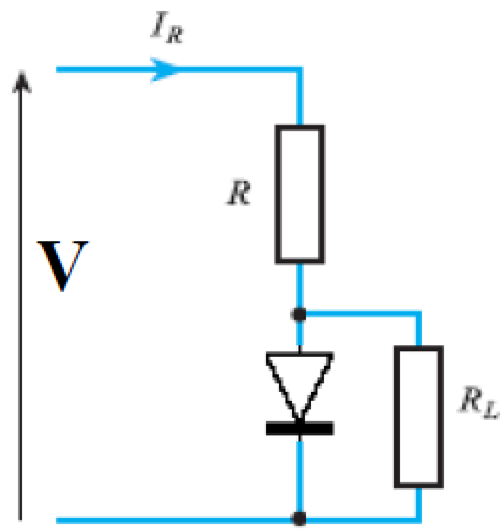
$$= \frac{1}{\frac{1-\omega^2R^2C^2}{j\omega RC}+3} = \frac{1}{3-j\frac{1-\omega^2R^2C^2}{\omega RC}}$$

For ω=1/RC B=V5/v3 is real → B=1/3

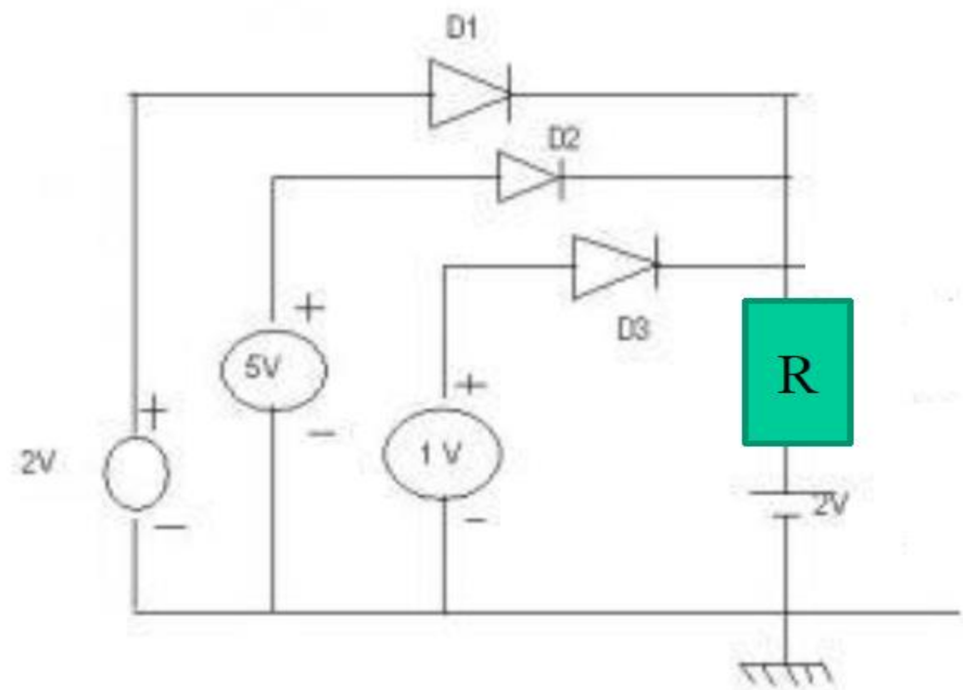
(c) AB=1, since B=1/3→ A=3 → R1/R2=2
Under these condition we have formed the Wien Oscillator

Problem 4 (1.5 points)

(a:0.5 points) The diode is ideal with forward conduction voltageVc. Calculate the current through the resistor R_L.



(b)



(b1: 0.5 point) Find which diode conducts current [*the diodes Di (i=1,2,3) are ideal with voltage for forward conduction Vc=0.5 V*].
(b2: 0.5 point) Calculate current via the resistor R

Solution

(a) Define $V_L = V \cdot \frac{R_L}{R+R_L}$ the potential on R_L if the Diode is absent.

- If $V_L < V_c \Rightarrow$ Diode is not conducting $\Rightarrow$

$I_{R_L} = V/R+R_L$

- If $V_L \geq V_c \Rightarrow$ Diode is conducting $\Rightarrow$

$I_{R_L} = \frac{V_c}{R_L}$

b1) Only D2 conducts. This is because if we denote with V the three input voltages (1, 2, 5 V) then the voltage difference V-2 only for D2 is larger than Vc (V-2 > Vc) to support forward conduction. (1 point)

b2) I=[(5-Vc)-2] /R = 2.5/R (1 point)

Problem 5 (1.5 points)

Design a synchronous counter that goes through the states (use J-K flip flops) 0, 1, 2, 4, 5, 6 shown below:

Before state			**After state**		
Q3	**Q2**	**Q1**	**Q3**	**Q2**	**Q1**
0	0	0			
0	0	1			
0	1	0			
1	0	0			
1	0	1			
1	1	0			

Q_{n-1}	Q_n	J	K
0	0	0	*
0	1	1	*
1	0	*	1
1	1	*	0

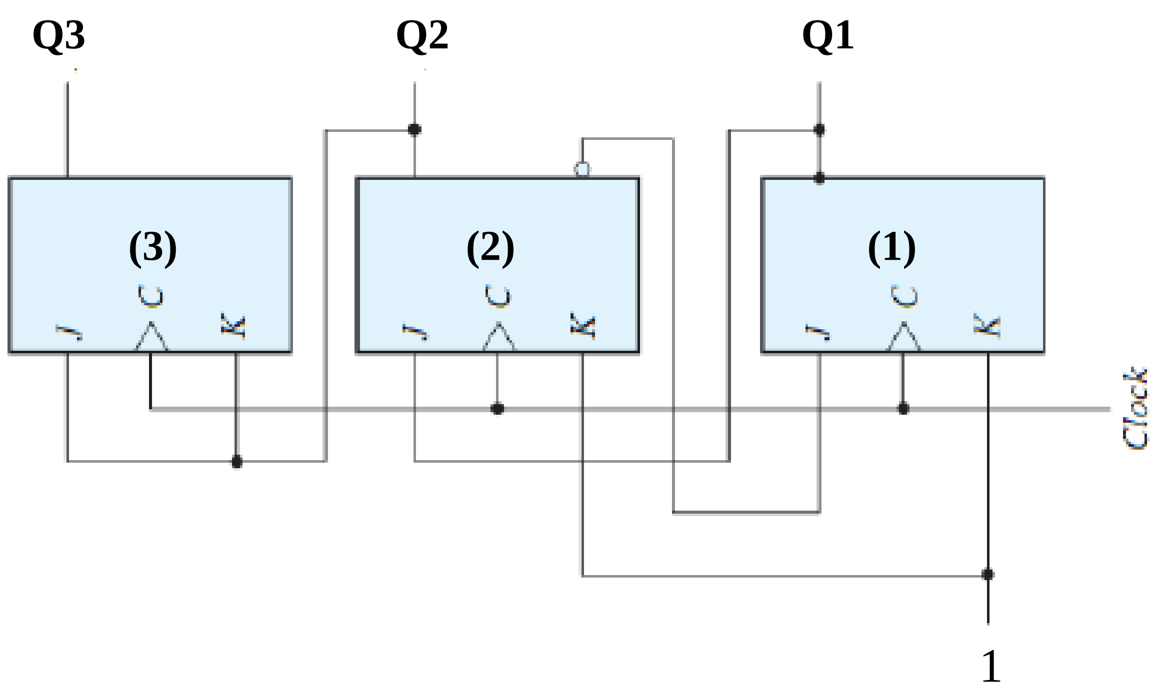
***: don't care**

J	K	Q_n
0	0	Q_{n-1}
0	1	0
1	0	1
1	1	$\overline{Q_{n-1}}$

Solution

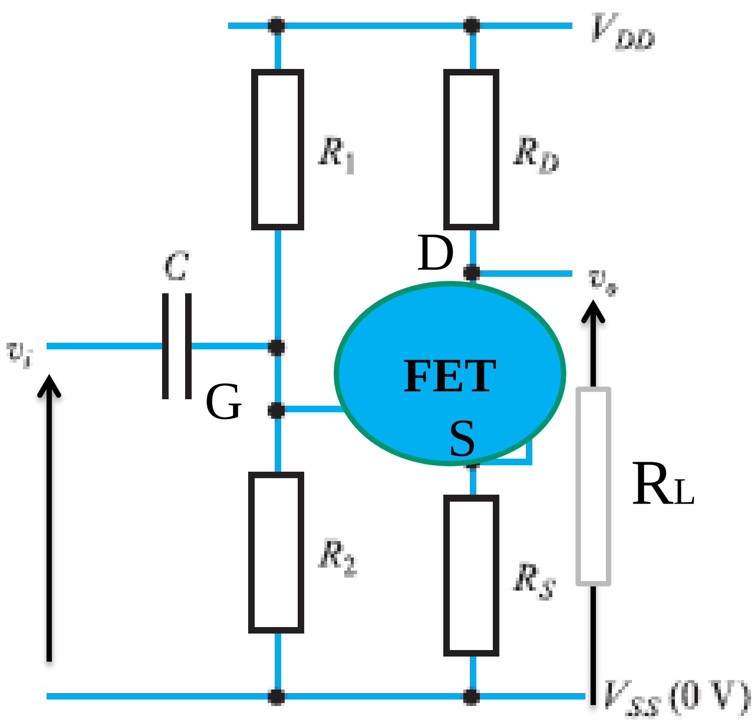
Before state			After state			Flip-Flop Inputs					
Q3	Q2	Q1	Q3	Q2	Q1	J3	K3	J2	K2	J1	K1
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	1	0	0	1	X	X	1	0	X
1	0	0	1	0	1	X	0	0	X	1	X
1	0	1	1	1	0	X	0	1	X	X	1
1	1	0	0	0	0	X	1	X	1	0	X

$J3=K3=Q2$, $J2=Q1$, $K2=1$, $J1=\overline{Q2}$, $K1=1$



Problem 5 (1.5 points)

An application of a small varying input signal υ_i leads to small variation of the gate, drain, and source potentials :



$$V'_G = V_G + \upsilon_g$$

$$V'_D = V_D + \upsilon_d$$

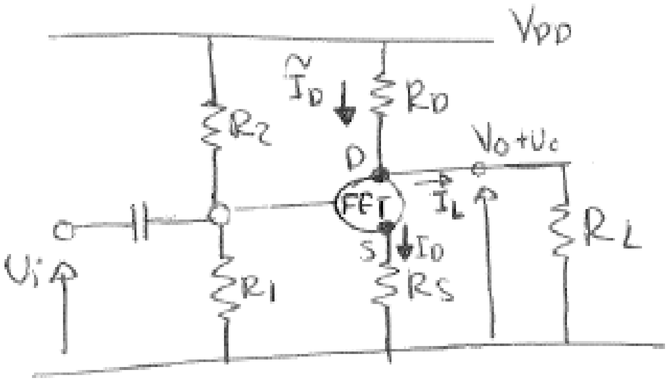
$$V'_S = V_S + \upsilon_s$$

If we connect a load resistor RL at output of the drain D (and the ground Vss=0), then show that the amplification ratio υ_o/υ_i is given by:

$$\frac{\upsilon_o}{\upsilon_i} = -\frac{g_m(R_D // R_L)}{1 + g_m R_S + [(R_D // R_L + R_S)/r_d]}$$

Solution

(1-method: first priciples analysis) *This is only for tough cookies!*



$v_g \equiv v_i$

$v_o \equiv v_d$

$i_d = g_m(v_g - v_s) + \frac{v_d - v_s}{r_d}$

$\tilde{I}_D = I_D + I_L$ (1)

$V_o = V_{DD} - \tilde{I}_D \cdot R_D \Rightarrow$ take a variation $\Rightarrow \delta \tilde{I}_D = i_d + i_L$

$\delta V_o = v_o = \delta V_{DD}^{\,0} - \delta \tilde{I}_D \cdot R_D \Rightarrow$

$v_o = -(i_d + i_L) R_D$ (2)

$V_o = I_L \cdot R_L \Rightarrow \delta V_o = v_o = \underbrace{\delta I_L}_{i_L} \cdot R_L \Rightarrow$

$v_o = i_L \cdot R_L \Rightarrow$ $i_L = v_o / R_L$ (3)

(2) & (3) $\Rightarrow v_o = -i_d R_D - v_o \frac{R_D}{R_L} \Rightarrow$

$\Rightarrow v_o \left[1 + \frac{R_D}{R_L}\right] = -i_d R_D$ (4)

we still need to eliminate in i_d the potential v_s

(1) $\Rightarrow \overbrace{\frac{V_{DD} - V_o}{R_D}}^{\tilde{I}_D} = \overbrace{\frac{V_S - 0}{R_S}}^{I_D} + \overbrace{\frac{V_o - 0}{R_L}}^{I_L}$, take a variation

$-\frac{\overbrace{\delta V_o}^{v_o}}{R_D} = \frac{\overbrace{\delta V_S}^{v_s}}{R_S} + \frac{\overbrace{\delta V_o}^{v_o}}{R_L} \Rightarrow -\frac{v_o}{R_D} = \frac{v_s}{R_S} + \frac{v_o}{R_L} \Rightarrow$

$\frac{v_s}{R_S} = -v_o\left[\frac{1}{R_D} + \frac{1}{R_L}\right] \Rightarrow$ $v_s = -v_o \frac{R_S}{R_{DL}}$, $R_{DL} = R_D \| R_L$ (5)

substitute in (4) from (5) the V_S and replace also $V_g = V_i$, $V_d = V_o$

$$V_o\left[1+\frac{R_D}{R_L}\right] = -\left[g_m\left(V_i+V_o\frac{R_S}{R_{DL}}\right) + V_o\frac{1+\frac{R_S}{R_{DL}}}{r_d}\right]R_D$$

$$V_o\left[\frac{1}{R_D}+\frac{1}{R_L}\right] = -g_m V_i - g_m V_o\frac{R_S}{R_{DL}} - V_o\frac{R_{DL}+R_S}{r_d R_{DL}}$$

$$\frac{V_o}{R_{DL}} + g_m V_o\frac{R_S}{R_{DL}} + V_o\frac{R_{DL}+R_S}{r_d R_{DL}} = -g_m V_i \Rightarrow$$

$$V_o\left[1+g_m R_S+\frac{R_{DL}+R_S}{r_d}\right]\frac{1}{R_{DL}} = -g_m V_i \Rightarrow$$

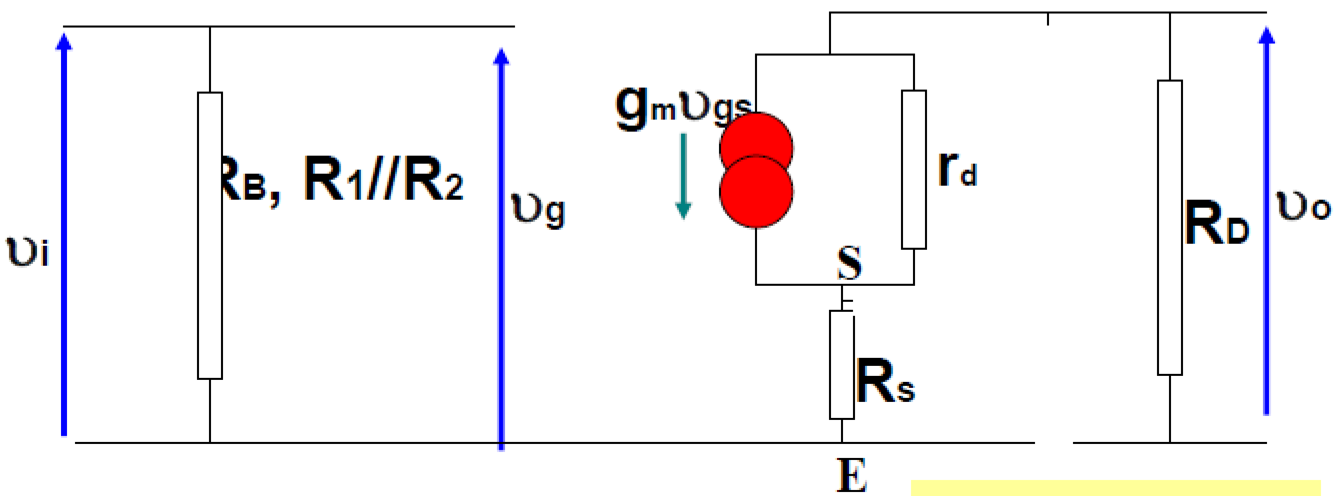
$$\boxed{\frac{V_o}{V_i} = -\frac{g_m R_{DL}}{1+g_m R_S+\frac{R_{DL}+R_S}{r_d}}}$$

Although this looks complicated, this is what is happening in reality!

you can extend this approach beyond first order perturbation theory- a limitation for method-2 as follows!.

(2-method: small signal cicuit) *This is for normal cookies!*

Replace in all shown bellow: RD with RD//RL . This is because in this design RD parallel with RL



Apply K-law for the current at points S & E

$$g_m \upsilon_{gs} + (\upsilon_o - \upsilon_s)/r_d - \upsilon_s / R_s = 0 \qquad (S)$$

$$(\upsilon_o / R_D) + (\upsilon_s / R_s) = 0 \qquad (E)$$

$$\upsilon_{gs} = \upsilon_g - \upsilon_s$$

$$g_m \upsilon_{gs} + (\upsilon_o - \upsilon_s)/r_d - \upsilon_s/R_s = 0 \quad \text{(S)}$$

$$(\upsilon_o/R_D) + (\upsilon_s/R_s) = 0 \quad \text{(E)}$$

$$\upsilon_{gs} = \upsilon_g - \upsilon_s$$

$$\Downarrow$$

$$\upsilon_g = \upsilon_i$$

Gain: $\upsilon_o/\upsilon_i = -g_m R_D/[1 + g_m R_s + (R_s + R_D)/r_d]$

To have the desired result replace : $R_D \rightarrow R_D//R_L$